

演習問題 4

学部： _____ 学科： _____

氏名： _____ 学籍番号： _____

問題 6. 次の行列の簡約化行列を求めなさい。

$$\begin{aligned}
 (1) \quad & \begin{bmatrix} 2 & -4 & 2 & -3 & 6 \\ -1 & 2 & -5 & -1 & 0 \\ 2 & -4 & -14 & -13 & 18 \\ -5 & 10 & -17 & 0 & -6 \end{bmatrix} \xrightarrow[r_4-5r_2]{r_3-r_1} \begin{bmatrix} 2 & -4 & 2 & -3 & 6 \\ -1 & 2 & -5 & -1 & 0 \\ 0 & 0 & \boxed{\text{ア}} & \boxed{\text{イ}} & \boxed{\text{ウ}} \\ 0 & 0 & \boxed{\text{エ}} & \boxed{\text{オ}} & \boxed{\text{カ}} \end{bmatrix} \\
 & \xrightarrow[r_3+2r_4]{r_1+2r_2} \begin{bmatrix} 0 & 0 & \boxed{\text{キ}} & \boxed{\text{ク}} & \boxed{\text{ケ}} \\ -1 & 2 & -5 & -1 & 0 \\ 0 & 0 & \boxed{\text{コ}} & \boxed{\text{サ}} & \boxed{\text{シ}} \\ 0 & 0 & \boxed{\text{エ}} & \boxed{\text{オ}} & \boxed{\text{カ}} \end{bmatrix} \xrightarrow[r_2 \times (-1)]{r_1+r_4} \begin{bmatrix} 0 & 0 & \boxed{\text{ス}} & \boxed{\text{セ}} & \boxed{\text{ソ}} \\ 1 & -2 & 5 & 1 & 0 \\ 0 & 0 & \boxed{\text{コ}} & \boxed{\text{サ}} & \boxed{\text{シ}} \\ 0 & 0 & \boxed{\text{エ}} & \boxed{\text{オ}} & \boxed{\text{カ}} \end{bmatrix} \\
 & \xrightarrow[r_2-\frac{5}{8} \times r_4]{r_2 \times \frac{1}{8}} \begin{bmatrix} 0 & 0 & \boxed{\text{ス}} & \boxed{\text{セ}} & \boxed{\text{ソ}} \\ 1 & -2 & \boxed{\text{タ}} & \boxed{\text{チ}} & \boxed{\text{ツ}} \\ 0 & 0 & \boxed{\text{コ}} & \boxed{\text{サ}} & \boxed{\text{シ}} \\ 0 & 0 & \boxed{\text{エ}} & \boxed{\text{オ}} & \boxed{\text{カ}} \end{bmatrix} \xrightarrow[r_4 \times \frac{1}{8}]{r_2 \times \frac{1}{8}} \begin{bmatrix} 0 & 0 & \boxed{\text{ス}} & \boxed{\text{セ}} & \boxed{\text{ソ}} \\ 1 & -2 & \boxed{\text{タ}} & \boxed{\text{チ}} & \boxed{\text{ツ}} \\ 0 & 0 & \boxed{\text{コ}} & \boxed{\text{サ}} & \boxed{\text{シ}} \\ 0 & 0 & \boxed{\text{テ}} & \boxed{\text{ト}} & \boxed{\text{ナ}} \end{bmatrix} \\
 & \xrightarrow{\text{並び替え}} \begin{bmatrix} 1 & -2 & \boxed{\text{タ}} & \boxed{\text{チ}} & \boxed{\text{ツ}} \\ 0 & 0 & \boxed{\text{テ}} & \boxed{\text{ト}} & \boxed{\text{ナ}} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad & \begin{bmatrix} 1 & 2 & -3 & 2 & 1 \\ 1 & 3 & -3 & 2 & 0 \\ 2 & 4 & -6 & 3 & 4 \\ 1 & 1 & -4 & 1 & 6 \end{bmatrix} \xrightarrow{r_3-2r_1} \begin{bmatrix} 1 & 2 & -3 & 2 & 1 \\ 1 & 3 & -3 & 2 & 0 \\ 0 & 0 & 0 & \boxed{\text{ア}} & \boxed{\text{イ}} \\ 1 & 1 & -4 & 1 & 6 \end{bmatrix} \xrightarrow[r_2-r_4]{r_1-r_4} \begin{bmatrix} 0 & \boxed{\text{ウ}} & \boxed{\text{エ}} & \boxed{\text{オ}} & \boxed{\text{カ}} \\ 0 & 0 & \boxed{\text{サ}} & \boxed{\text{シ}} & \boxed{\text{ス}} \\ 0 & 0 & 0 & \boxed{\text{ア}} & \boxed{\text{イ}} \\ 1 & 1 & -4 & 1 & 6 \end{bmatrix} \\
 & \xrightarrow{r_2-2r_1} \begin{bmatrix} 0 & \boxed{\text{ウ}} & \boxed{\text{エ}} & \boxed{\text{オ}} & \boxed{\text{カ}} \\ 0 & 0 & \boxed{\text{サ}} & \boxed{\text{シ}} & \boxed{\text{ス}} \\ 0 & 0 & 0 & \boxed{\text{ア}} & \boxed{\text{イ}} \\ 1 & 1 & -4 & 1 & 6 \end{bmatrix} \xrightarrow{r_4-r_1} \begin{bmatrix} 0 & \boxed{\text{ウ}} & \boxed{\text{エ}} & \boxed{\text{オ}} & \boxed{\text{カ}} \\ 0 & 0 & \boxed{\text{サ}} & \boxed{\text{シ}} & \boxed{\text{ス}} \\ 0 & 0 & 0 & \boxed{\text{ア}} & \boxed{\text{イ}} \\ 1 & \boxed{\text{セ}} & \boxed{\text{ソ}} & \boxed{\text{タ}} & \boxed{\text{チ}} \end{bmatrix} \\
 & \xrightarrow{r_1+r_2} \begin{bmatrix} 0 & \boxed{\text{ツ}} & \boxed{\text{テ}} & \boxed{\text{ト}} & \boxed{\text{ナ}} \\ 0 & 0 & \boxed{\text{サ}} & \boxed{\text{シ}} & \boxed{\text{ス}} \\ 0 & 0 & 0 & \boxed{\text{ア}} & \boxed{\text{イ}} \\ 1 & \boxed{\text{セ}} & \boxed{\text{ソ}} & \boxed{\text{タ}} & \boxed{\text{チ}} \end{bmatrix} \xrightarrow{r_4-5r_2} \begin{bmatrix} 0 & \boxed{\text{ツ}} & \boxed{\text{テ}} & \boxed{\text{ト}} & \boxed{\text{ナ}} \\ 0 & 0 & \boxed{\text{サ}} & \boxed{\text{シ}} & \boxed{\text{ス}} \\ 0 & 0 & 0 & \boxed{\text{ア}} & \boxed{\text{イ}} \\ 1 & 0 & \boxed{\text{ニ}} & \boxed{\text{ヌ}} & \boxed{\text{ネ}} \end{bmatrix} \\
 & \xrightarrow[r_4+5r_3]{r_2-r_3} \begin{bmatrix} 0 & \boxed{\text{ツ}} & \boxed{\text{テ}} & \boxed{\text{ト}} & \boxed{\text{ナ}} \\ 0 & 0 & \boxed{\text{ノ}} & \boxed{\text{ハ}} & \boxed{\text{ヒ}} \\ 0 & 0 & 0 & \boxed{\text{ア}} & \boxed{\text{イ}} \\ 1 & 0 & \boxed{\text{フ}} & \boxed{\text{ヘ}} & \boxed{\text{ホ}} \end{bmatrix} \xrightarrow{\text{簡約化すると}} \begin{bmatrix} 1 & 0 & 0 & 0 & \boxed{\text{マ}} \\ 0 & 1 & 0 & 0 & \boxed{\text{ミ}} \\ 0 & 0 & 1 & 0 & \boxed{\text{ム}} \\ 0 & 0 & 0 & 1 & \boxed{\text{メ}} \end{bmatrix}
 \end{aligned}$$

$$(3) \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \end{bmatrix} \xrightarrow{\text{簡約化行列は}} \begin{bmatrix} 1 & 0 & \boxed{\text{ア}} & \boxed{\text{イ}} & \boxed{\text{ウ}} \\ 0 & 1 & \boxed{\text{エ}} & \boxed{\text{オ}} & \boxed{\text{カ}} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$(4) \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \\ 1 & 1 & 1 & -3 \end{bmatrix} \xrightarrow{\text{簡約化行列は}} \begin{bmatrix} 1 & \boxed{\text{ア}} & \boxed{\text{イ}} & \boxed{\text{ウ}} \\ 0 & 1 & \boxed{\text{エ}} & \boxed{\text{オ}} \\ 0 & 0 & 1 & \boxed{\text{カ}} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$(5) \begin{bmatrix} 1 & 2 & 1 & 0 \\ 1 & 3 & -1 & 3 \\ 2 & -1 & 2 & 5 \\ 5 & 6 & 3 & 2 \\ 3 & 2 & 1 & 2 \end{bmatrix}$$